# CLOSURE PROPERTY

**Words to be Learned**

**Exercise 1**

*Read these international words and try to guess their meaning.*

natural ['næt∫(ə)r(ə)l] *a*, unique [ju:'ni:k] *a,* uniqueness [ju:'ni:knıs] *n*, 'general [ʤenərəl] *a*, 'element *n,* axiom ['æksıəm] *n,* summarize ['sлməraız] *v*, definition [**ֽ**defı'nı∫(ə)n] *n*, 'practically *adv*, pro'duce *v.*

**Exercise 2**

*Repeat after the teacher.*

|  |  |
| --- | --- |
| concern [kən'sɜ:n] *v* касаться, иметь отношение к  be concerned with заниматься, быть заинтересованным  property ['prɔpətı] *n* свойство  closure ['kləuʒə] *n* замкнутость  follow ['fɔləu] *v* следовать за  statement ['steıtmənt] *n* утверждение, заявление  state *v* утверждать  notice ['nəutıs] *v* замечать, обращать внимание  exist [ıg'zıst] *v* существовать  existence [ıg'zıstəns] *n* существование  both [bəuθ] оба  imply [ım'plaı] *v* 1. заключать в себе; 2. иметь в виду, означать, предполагать  define [dı'faın] *v* определять, давать определение  instead (of) [ın'sted] вместо  asterisk ['æstərısk] *n* звездочка *(в обозначениях)*  pair [peə] *n* пара; *v* удваивать  consider [kən'sıdə] *n* 1. считать, полагать; 2. рассматривать | necessarily ['nesəs(ə)r(ə)lı], [**ֽ**nesə'ser(ə)lı] *adv* обязательно  necessary ['nesəs(ə)rı] *a* обязательный  true [tru:] *a* истинный, справедливый *(для)*  appear [ə'pıə] *v* 1. появляться; 2. казаться  several ['sevr(ə)l] несколько  denote [dı'nəut] *v* обозначать  impossible [ım'pɔsıbl] *a* невозможный  happen ['hæp(ə)n] *v* иметь место, произойти  point out *v* отмечать  hence следовательно  thus таким образом  accept [ək'sept] *v* принимать, признавать  proof [pru:f] *n* доказательство  prove [pru:v] *v* доказывать  capital ['kæpıt(ə)l] *a* заглавный, важный  allow [ə'lau] *n* разрешать, позволять  should [∫ud] следует, должен  (be) able [eıbl] быть способным, мочь |

**Notes**

1. following are other examples – далее следуют другие примеры
2. both ... and – и ... и, как ... так и
3. in general – вообще
4. as follows – следующим образом
5. *A* capital – заглавное *A*
6. it should be pointed out – следует отметить

**Exercise 3**

*Listen and repeat. Remember what letters and letter combinations the sounds* [ı], [aı], [eı], [ɔı] *are expressed by.*

[ı] – duty, till, ready, did, language, determine, examine, knowledge, damage, daily, symbol, cynic, film;

[aı] – right, try, mile, light, fly, by, mild, hide, flight, bind, dry, analyse, why, wine;

[eı] – main, game, eight, day, straight, agent, against, grey, stay, way, fame, rain, away, play;

[ɔı] – boy, oil, spoil, loyal, employ, point, foil, avoid.

**Exercise 4**

*Comment on the speaker's statements. Follow the model.*

Sp.: She *cannot* give you the journal now. (next week)

St.: I *hope,* she *will be able to* do it next week.

1. They could not operate the machine well. (in some time) 2. She cannot translate scientific articles. (next year) 3. I cannot obtain enough information. (in a few days) 4. They cannot produce such complicated computers now. (in future) 5. He could not solve this complicated problem. (with your help)

**Exercise 5**

**a)** *Ask a question following the model below.*

Sp.: I cannot use these data now. (in your paper)

St.: *Will you be able to* use these data in your paper?

1. He cannot leave Moscow now. (after his exams) 2. She could not visit her home town in September. (during her winter holidays) 3. They could not get tickets for the theatre on Monday. (on Saturday). 4. He can not attend lectures now because he is ill. (next month)

**b)** *Comment on the speaker's statements.*

Sp.: I am not able to do research now. (last year)

St.: I *suppose* you were able to do research last year.

1. She is not able to speak English well now. (a few years ago) 2. He is not able to work at home today. (last Sunday) 3. I am not able to discuss it with my science adviser at present. (last Tuesday) 4. We are not able to help her at the moment. (some time ago). 5. They are not able to play chess tonight. (last night)

**Exercise 6**

*Answer these questions.*

Sp.: Were you allowed to change the program? (Yes, no)

St.: Yes, I was allowed to change the program. No, I was not allowed to change the program.

**a)** 1. Were you allowed to leave early? (No) 2. Were they allowed to check the work of the power-station? (Yes) 3. Was he allowed to occupy the lab for today? (Yes) 4. Were the students allowed to join the construction team in August? (No) 4. Are you allowed to take part in this investigation? (Yes) 5. Will the student be allowed to run this machine by himself? (No)

Sp.: When are you to meet? (after classes)

St.: We are to meet after classes.

**b)** 1. When is he to see his doctor? (tomorrow) 2. What is she to do with her paper? (give it to me) 3. When are you to speak to your science adviser? (after his lecture) 4. When was she to present her abstract (as soon as possible) 5. What were you to show that foreign scientist? (our recent data)

**Exercise 7**

*Answer these questions.*

Do you have to change the place of the meeting? (the time)

We have to change the time of the meeting.

1. Are the students to study the uniqueness property at the lesson? (the closure) 2. Was he able to give the proof? (the general definition) 3. Were the scientists to consider the plan of their future work at the conference? (the results of the recent experiment) 4. Did you have to discuss one of the articles? (both) 5. Were they able to obtain any data? (all the necessary data) 6. Does he have to summarize the results of the research work of the whole department? (of our laboratory) 7. Are you to begin your work now? (in a few days) 8. Ought you to tell everybody why you changed your mind? (my mother) 9. Should everything be done tomorrow? (today) 10. Were you able to notice the difference between these two methods? (their similarity)

**Exercise 8**

*Ask questions making use of the given question words.*

1. She had to agree with both of them. (why) 2. They were to learn the third law of mechanics. (when) 3. He is to come at four instead of five o'clock. (when) 4. Properties of existence and closure should be studied at the next lesson. (what) 5. You will not be able to obtain all the necessary books. (why) 6. This fact should be pointed out to them. (by whom) 7. There are various definitions of this property. (how many) 8. This new method has to be accepted as soon as possible. (why) 9. The property of existence is implied in the following definition. (which) 10. Their experiment was considered in an article in the physical journal. (when) 11. The proof of this problem is not very complicated. (what) 12. They were not allowed to begin research. (why)

**Exercise 9**

*Write the following sentences in the negative form.*

1. He will be able to notice the difference. 2. They should be concerned with the application of the new system. 3. We have to consider some of their statements at present. 4. He has to give us all the details of the future experiment. 5. We have to accept his words without proof. 6. It had to happen again. 7. These pairs of natural numbers are to be multiplied. 8. You ought to discuss both of these properties now. 9. They were able to discover another fundamental law of nature. 10. You have to learn two foreign languages while you are taking a postgraduate course.

**Exercise 11**

*Practise reading these words.*

empty, hid, determine, night, blind, sight, play, steady, ink, toy, examine, plain, plane, mist, mild, coil, hit, boil, average, win, wind, aim, nice, again, faint, stay, my, soil, grey, coin, why, language, hide, damage, mail, male, milt, might, eighty.

**Exercise 12**

*Name the predicate in the sentences below. Translate the sentences.*

1. It is necessary to summarize all the known facts. 2. The asterisk denotes a certain operation. 3. They were allowed to introduce their method. 4. We shall be able to consider the existing situation in detail. 5. You have to see both of these students and ask them about their plans. 6. This definition cannot be accepted. 7. His graduation paper is to be presented in January. 8. The problems considered appeared very important. 9. Closure uniqueness and existence properties were being discussed by the students during the lesson. 10. What are you trying lo imply?

**Exercise 13**

*Read the text below. Give a short outline of the text in Russian.*

**Closure Property**

In this lesson we shall be concerned with the closure property.

If we add two natural numbers, the sum will also be a natural number. For example, 5 is a natural number and 3 is a natural number. The sum of these two numbers, 8, is also a natural number. Following are other examples in which two natural numbers are being added and the sum is another natural number. 19+4=23 and only 23; 6+6=12 and only 12; 1429+357=1786 and only 1786. In fact, if you add any two natural numbers, the sum is again a natural number. Because this is true, we say that the set of natural numbers is closed under addition.

Notice that in each of the above equations we were able to name the sum. That is, the sum of 5 and 3 exists, or there is a number which is the sum of 19 and 4. In fact, the sum of any two numbers exists. This is called the existence property.

Notice also that if you are to add 5 and 3, you will get 8 and only 8 and not some other number. Since there is one and only one sum for 19+4, we say that the sum is unique. This is called the uniqueness property.

Both uniqueness and2 existence are implied in the definition of closure.

Now, let us state the closure property of addition.

If *a* and *b* are numbers of a given set, then *a+b* is also a number of that same set. For example, if *a* and *b* are any two natural numbers, then *a+b* exists, it is unique, and it is again a natural number.

If we use the operation of subtraction instead of the operation of addition, we shall not be able to make the statement we made above. If we are to subtract natural numbers, the result is sometimes a natural number, and sometimes not. 11–6=5 and 5 is a natural number, while 9–9=0 and 0 is not a natural number.

Consider the equation 4–7=*n.* We shall not be able to solve it if we must have a natural number as an answer. Therefore, the set of natural numbers is not closed under subtraction.

What about the operation of multiplication? Find the product of several pairs of natural numbers. Given two natural numbers, is there always a natural number which is the product of the two numbers?

Every pair of natural numbers has a unique product which is again a natural number. Thus the set of natural numbers is closed under multiplication.

In general, the closure property may be defined as follows4: if *x* and *у* are any elements, not necessarily the same, of set *A (A* capital)5 and \* (*asterisk*) denotes an operation \*, then set *A* is closed under the operation asterisk if (*x\*y*)is an element of set *A.*

To summarize, we shall say that there are two operations, addition and multiplication, for which the set of natural numbers is closed. Given any two natural numbers *x* and *y*, *x+y* and *x*×*y* are again natural numbers. This implies that the sum and the product of two natural numbers exists. It so happens that with the set of natural numbers (but not with every mathematical system) the results of the operations of addition and multiplication are unique.

It should be pointed out6 that it is practically impossible to find the sum or the product of *every* possible pair of natural numbers. Hence, we have to accept the closure property without proof, that is, as an axiom.

**Exercise 20**

*Listen and repeat. Guess the meaning of the italicized words.*

nature–*natural–naturally;* actually–*an act*–*to act;* true–truth–*truthful;* special–*specially*–*specialize;* to notice–*noticeable;* to imply–*implication;* to consider–*consideration;* to solve–*solution;* general–*generally*–*generalize;* necessary–*unnecessary;* impossible–*possible*–*possibility;* axiom–*axiomatic.*

**Exercise 14**

*Answer the following questions. Make use of the model below.*

Sp.: Was he able to give a lot of examples? (just a few)

St.: No, he was not. He was able to give just a few examples.

1. Did you have to rise early on Sunday? (at eight o'clock) 2. Were you to meet in the morning? (in the afternoon) 3. Was he able to provide them with all the necessary data? (very little) 4. Will they have to make a lot of changes? (just a few) 5. Are students able to work in the lab on Sunday? (only on week-days) 6. Did you have to write a lot of exercises for your homework? (only two) 7. Will they be able to visit all the places of interest in this town? (only the ancient part of the town) 8. Ought he to see a doctor? (stay in bed (*полежать в постели*) for a couple of days) 9. Are you to discuss the details? (the general situation) 10. Do they have to speak French at this international conference? (English)

**Exercise 15**

*Ask a general question about each one of his statements.*

1. This statement is not true. 2. They are going to discuss the properties of uniqueness and existence during the lesson. 3. I easily noticed the difference. 4. Both of the brothers came in due time. 5. This definition is rather long. 6. He is to come today instead of tomorrow. 7. This information is being considered by the head of our department. 8. We were able to find several books by this ancient author in his library. 9. This operation is denoted by an asterisk. 10. He has to accept this appointment at the moment.

**Exercise 16**

*Listen to the questions about the text and write down your answers with the help of a + or a* – *sign.*

1. Will the sum of two natural numbers be again a natural number? 2. If any two natural numbers are being added, is the sum again a natural number? 3. Is the set of natural numbers closed under addition? 4. Is the statement of closure one of the special properties of division? 5. Are we able to name the sum in the equations 19+4=23 and 6+6=12? 6. Does the sum of the above equations exist? 7. Are uniqueness and existence implied in the definition of closure? 8. If one natural number is being subtracted from another natural number, is the difference always a natural number? 9. Can the equation 4–7=*n* be solved if we are to have a natural number as an answer? 10. Is the set of natural numbers closed under subtraction? 11. Does every pair of natural numbers have a unique product? 12. Is the set of natural numbers closed under division? 13. Are we able to find practically the sum or the product of *every* possible pair of natural numbers? 14. Do we accept the closure property as an axiom?

**Exercise 17**

*Arrange the words according to the parts of speech they belong to.*

length, usage, knowledge, specify, minimize, actively, productive, productivity, primitive, effectiveness, symbolic, measure, producer, generalize, generalization, activity, appearance, acceptable, impossibility, follower, useful, helpless.

**Exercise 18**

*Disagree with the following statements.*

His words *had to* be accepted without proof.

No, his words did not have to be accepted without proof.

1. They should discuss the property of closure at the next lesson. 2. She follows her sister's example. 3. This important news is being considered. 4. It is necessary to remember all these laws. 5. There exist some new machines of this type. 6. The given element may be denoted by the same symbol. 7. They will be able to use some symbols instead of words. 8. You ought to accept everything he says as an axiom. 9. The information produced showed interesting results. 10. You are to change the time and the place of the meeting.

**Exercise 19**

**а)** *Read the text below and translate it;*

**b)** *Entitle the text.*

From the above you should remember that the result of multiplication is called a product, and that the numbers to be multiplied are called factors. When you write 6×3=18 it means that you write number 18 as a product of two whole number factors. Another pair of whole-number factors will be 9 and 2. Since 9×2=18 will you be able to name other factors of 18?

In some cases when we have to name a whole number in factored form, more than (более, чем) two factors can be used. We can, for example, name 60 as the product of 3 factors.

Since multiplication is associative [ə'səu∫jətıv], we know that (3×4)×5=3×4×5=3×(4×5). We may also write that 60 = 3×4×5; 60 = 3×5×4 and so on.

In each of the above equations the same set of factors is used, namely, 3, 4 and 5. Regardless of the order in which they are written, 3, 4 and 5 should be considered just one set of three factors of 60. Also 60 can be written as the product of four factors as shown in the equation 60 = 3×2×2×5. In some of your exercises you 'probably (вероятно) noticed that some factors you used could be 'factored further (*далее*) and others could not.

In the equation 18=6×3, the factor 6 can in turn be written as 3×2. If you do this, you will get 18 = 2×3×3. None of these three factors can be written in factored form if you do not use 1 as a factor. Hence 2×3×3 is the form containing the smallest (*наименьший*) factors of 18.

You already (уже) know that every whole number has 1 and itself (*себя*) as factors. Some such numbers have only itself and 1 as factors. Since its only factors are 1 and 5, we say that 5 is such a number.

The same may by said about 7 or 11. Numbers like 5, 7, or 11 are called prime numbers.

A whole number is called a prime number or just a prime if it meets the following conditions (*удовлетворяет следующим требованиям*): a) It is greater than 1; b) Its only factors are itself and 1.

Any whole number, with the exception of ([ık'sep∫(ə)n] (*за исключением*) 0 or 1 which is not a prime number, is called a 'composite number, or just a composite.

**Exercise 20**

*Get ready to discuss the text in class. Ask your class-mates a few questions and be prepared to answer theirs.*

**Exercise 21**

*Read the words and give the Russian equivalents of the words in italics.*

whole–*'wholly;* name–*to name*–*nameless*–*namely;* mean–*meaning– meaningful–meaningless;* a factor–*to factor–'factorable;* to associate–associative–association; 'order *n–dis'order,* 'previous– *'previously;* 'probably–*'probable–probability*–*im'probable;* exception– *exceptional;* 'composite–*compo'sition–to com'pose;* 'practical–*im'practical;* 'necessary–*unnecessary;* to prove–*to dis'prove;* true–*un'true;* closed–*'closure*–*to dis'close;* to 'notice–*un'noticed.*

**Exercise 22**

*Give an adequate translation of each sentence below.*

1. Нам предстоит рассмотреть множество чисел. 2. Вам следует помнить, что результат умножения называется произведением. 3. Числа, которым предстоит быть помноженными, называются сомножителями. 4. Давайте согласимся называть 6 и 3 одной парой сомножителей числа 18. 5. Вам следует помнить, что умножение ассоциативно. 6. В каждом из вышеприведенных уравнений нам дан один и тот же ряд сомножителей. 7. Порядок, в котором они должны быть записаны, не существен. 8. Вы сможете разложить данное число дальше. 9. Сумма двух натуральных чисел есть тоже натуральное число. 10. Поскольку это справедливо, мы говорим, что это замкнутое множество. 11. Свойство замкнутости есть одно из свойств сложения. 12. По существу сумма любых двух чисел существует. 13. Рассмотрим уравнение 4−7=*п.* 14. Следовательно, множество натуральных чисел не является замкнутым при вычитании. 15. Вообще свойство замкнутости может быть определено следующим образом. 16. Следовательно, мы принимаем свойство замкнутости как аксиому без какого-либо доказательства.